H M CA

Solved Paper 2013

Part A

1. On a straight road XY, 100 m long, five heavy stones are placed 2 m apart beginning at the end X. A worker, starting at X, has to transport all the stones to Y, by carrying only one stone at a time. The minimum distance he has to travel is

(a) 422 m (b) 480 m (c) 744 m (d) 860 m

2. If $a^b c^a \quad abca$, where all a, b and c are integers, then which of the following is true?

(a) $a \quad 1$ (b) $b \quad 4$
(c) $c \quad 9$ (d) None of these

3. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to the longer side is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

4. Four cows are tethered at four corners of a square plot of side 14 m, so that the adjacent cows can just reach one another. There is a small circular pond of area $20\,\text{m}^2$ at the centre. The area left ungrazed is

(a) $22\ \text{m}^2$ (b) $42\ \text{m}^2$ (c) $84\ \text{m}^2$ (d) $168\ \text{m}^2$

5. How many number of times will the digit 7's be written, when listing the integers from 1 to 1000?

(a) 272 (b) 252 (c) 300 (d) 304

6. There are 10 positive real numbers $n_1 \quad n_2 \quad n_3 \qquad n_{10}$. How many triplets of these numbers $(n_1, n_2, n_3), (n_2, n_3, n_4), \ldots$ can be generated, such that in each triplet the first number is always less than the second number and the second number is always less than the third number?

(a) 45 (b) 90 (c) 120 (d) 180

7. There were two women amongst other men who took part in a chess tournament. Every participart played two games with every other participart. The number of games which only men played was exactly 104 games more than those played which involved a women. The total number of participarts is

(a) 11 (b) 14 (c) 13 (d) 15

8. It takes 6 technicians a total of 10 h to install a new equipment from scratch, with each working at the same rate. If six technicians start to install the same equipment at 11:00 am, and one technician per hour is added beginning at 5:00 pm, at what time will the equipment installation be complete?

(a) 6:40 pm (b) 7:00 pm (c) 7:20 pm (d) 8:00 pm

9. In how many ways is it possible to choose a white square and a black square on a chess board, so that the square must not lie in the same row or column?

(a) 56 (b) 896 (c) 60 (d) 768

10. Gopal's shop sells small cookies in boxes of different sizes. The cookies are priced at ` 2 per cookie up to 200 cookies. For every additional 20 cookies, the price of the whole lot goes down by 10 paise per cookie. What should be the maximum size of the box (in terms of number of cookies it can hold) that would maximize the revenue?

(a) 240 (b) 300
(c) 400 (d) None of these

Directions (Q. Nos. 11-12) Read the following information carefully to answer the questions that follow.

8 trees, viz. mango, guava, papaya, pomegranate, lemon, banana, raspberry and apple are planted in two rows of four each aligned East-West. Lemon is between mango and apple but just opposite to guava. Banana is either at the end of a row and is just immediately to the right of guava or banana is just next to guava. Raspberry is at the end of a row and mango is at the other end of the opposite row.

11. Which of the following is always true?

(a) Papaya is just next to apple
(b) Raspberry is either to the left or to the right of pomegranate
(c) Apple is just next to lemon
(d) Pomegranate is diagonally opposite to banana

12. Which of these is directly opposite to banana?

(a) Mango (b) Papaya
(c) Pomegranate (d) None of these

13. 3 small pumps and a large pump are filling a tank. Each of the three small pumps works at $\frac{2}{3}$ rd the rate of the large pump. In what fraction of the time that all 4 pumps working together will fill the tank in comparison to the time taken by the large pump alone?

(a) $\frac{4}{7}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Directions (Q. Nos. 14-15) Read the following information carefully to answer the questions that follow.
In the English alphabet there are 11 symmetric letters that appear the same when looked at in a mirror. Other 15 letters in the alphabet are asymmetric letters.

14. How many four letter computer passwords can be formed using only the symmetric letters (on repetition allowed)?

(a) 7920 (b) 330 (c) 14640 (d) 419430

15. How many three letter computer passwords (no repetition allowed) can be formed with atleast one symmetric letter?

(a) 990 (b) 2145 (c) 12870 (d) 15600

16. Which letter in the word 'RUTHLESS' has a position from the beginning of the word that is half as much as its position when seen in the alphabet?

(a) L (b) R (c) E (d) H

17. A wire of some length is bent in circular form and has an area of 308 sq cm. If the same length of wire is straightened out and bent in the form of a square, the approximate area of the square (in sq cm) may be

(a) 242 (b) 308 (c) 121 (d) 69.29

18. Consider a square circumscribed by a circle with a radius of 4 units. The area of the square (in sq units) is

(a) $16\sqrt{2}$ (b) 32 (c) 16 (d) 64

19. A man returns after shooting and catching birds in his bag. He was asked how many birds he had in his bag. He said, 'They are all house sparrows but six, they are all pigeons but six, and all doves but six.' The number of birds he had in all were?

(a) 18 (b) 9 (c) 36 (d) 27

20. Suppose an ant is placed on one corner of a sugar cube, which has equal sides of 1.5 cm each. If the ant may walk only along the edges of the cube. What is the maximum distance the ant may walk on the cube without retracing its path?

(a) 12 cm (b) 10.5 cm (c) 18 cm (d) 13.5 cm

21. When the big hand of the clock is exactly at the 12 o'clock position, an ant starts to crawl in a counter clockwise direction from the 6 o'clock position at a constant speed. On reaching the big hand of the clock, the ant turns around and at the same speed, starts to crawl, in the opposite direction. Exactly 45 min after the first meeting with the big hand the ant crosses the big hand for the second time and dies. How long has the ant been crawling?

(a) 51 min (b) 1 h and 9 min
(c) 54 min (d) 1 h and 21 min

22. One-third of Shrihari's marks in Math equal half of his English marks. Shrihari noticed that in these two subjects his marks totaled 150. What did Shrihari score in English?

(a) 60 (b) 30 (c) 15 (d) 90

23. The sum of two digits of a number is 15. If 9 is added to the number, then the digits get reversed. Which of the following is true about the number?

(a) The number has the two digits separated by a difference of one
(b) The number is divisible by 6
(c) The number is divisible by 3
(d) The number is divisible by 9

24. An ant is at a point P in a planar square field. It was observed that P is 13 ft from the corner A, and 17 ft from corner B (diagonal to A), and finally 20 ft from a third corner. The area (in sq ft) of the field is

(a) 89 (b) 369 (c) 231 (d) 169

25. Consider the ABC with sides AB 20 cm, AC 11 cm, and BC 13 cm. Then, the length in cm of the diameter of the semi-circle inscribed within ABC, which lies on AB and has sides AC and BC as tangents is given by

(a) 11 (b) 10 (c) 9 (d) 10.5

Part B

26. The derivative of function $f(x)$, where

$$f(x) \quad \begin{matrix} \frac{\sin x^2}{x} & ; x \quad 0 \\ 0 & ; x \quad 0 \end{matrix}$$

(a) does not exist at $x \quad 0$
(b) exists at $x \quad 0$ and is also continuous
(c) exists at $x \quad 0$ but is not continuous
(d) None of the above

27. Use the notation $|x|$ to denote the absolute value of x, and $|x|$ denotes the largest integer smaller or equal to x. For a, an integer, such that $a \quad 1$ let $f(x)$ be given by

$$f(x) \quad \begin{matrix} ||x \quad 1|| \quad 1|, & x \quad a \\ |x|, & x \quad a \end{matrix}$$

Which of the following is the set of points of discontinuity for the function $f(x)$?

(a) All integers a (b) All integers a
(c) All integers a (d) a

28. Consider the function

$$f(x, y) \quad \begin{matrix} \frac{xy}{x^2 \quad y^2} & ; (x, y) \quad (0, 0) \\ 0 & ; (x, y) \quad (0, 0) \end{matrix}$$

Which of the following is not true for this function?

(a) f is continuous along the line $x \quad 0$
(b) f is constant along the line $y \quad x$
(c) f is continuous along the line $y \quad 0$
(d) f is not continuous at $(0, 0)$

Directions (Q. Nos. 29-30) Based on the algorithm given below.

Step 0 : START
Step 1 : N : 0, C : 0, FLAG : FALSE
Step 2 : INPUT TABLE $A[1 \quad 10]$
Step 3 : For I 1 to 10 Do
if $(A[I] \quad 1)$ then
$N : \quad N \quad 1$
If (FLAG FALSE) THEN
$C \quad I$
FLAG TRUE
END IF
END IF
END FOR
Step 4 : OUTPUT N, C
Step 5 : END
Given that A $[1 \quad 10]$ 7, 0, 0, 1, 1, 0, 3, 1, 9, 10

29. What are the values of N and C?

(a) $N \quad 4, C \quad 10$
(b) $N \quad 4, C \quad 4$
(c) $N \quad 3, C \quad 4$
(d) $N \quad 3, C \quad 10$

30. What are the values of N and C, if the statement 'FLAG TRUE' is removed from the algorithm?

(a) $N \quad 4, C \quad 10$
(b) $N \quad 3, C \quad 8$
(c) $N \quad 4, C \quad 8$
(d) $N \quad 3, C \quad 10$

31. The nth element of a series is represented as $X_n \quad (\quad 1)^n X_{n \quad 1}$. If $X_0 \quad x$ and $x \quad 0$, then which of the following is always true?

(a) X_n is positive, if n is even
(b) X_n is positive, if n is odd
(c) X_n is negative, if n is even
(d) None of the above

32. If n is an integer, then which of the following is true about the sum $S \quad 2^n \quad 3^n$?

(a) S is always the square of a rational number
(b) S is the square of a rational number, provided n is even
(c) S is never the square of a rational number
(d) S is the square of a rational number, provided n is odd

33. If $f(x) \quad \log \frac{1 \quad x}{1 \quad x}$, then $f(x) \quad f(y)$ is equal to

(a) $f \frac{x \quad y}{1 \quad xy}$ (b) $f\,(x \quad y)$
(c) $(x \quad y)\, f \frac{1}{1 \quad xy}$ (d) $\frac{f(x) \quad f(y)}{1 \quad xy}$

34. In an equilateral triangle, let h, be the height and r be the radius of the circumcircle. Then, the ratio $r : h$, is

(a) $2 : 3$ (b) $2 \quad : \sqrt{3}$
(c) $\sqrt{3} : 1$ (d) $\quad : \sqrt{5}/2$

35. Let denote an empty set. The power set of the empty set, is

(a) { } (b) { , { }}
(c) (d) None of these

36. Let A and B be two non-empty sets with cardinality p and q, respectively. Then the total number of relations that can be defined from the set A to set B is

(a) 2^p (b) 2^{pq}
(c) 2^q (d) p or q or pv

37. Consider the relation f defined by

$f(x) \quad \begin{matrix} x^2; & 0 \quad x \quad 3 \\ 3x; & 3 \quad x \quad 10 \end{matrix}$ and the relation g defined by

$g(x) \quad \begin{matrix} x^2; & 0 \quad x \quad 2 \\ 3x; & 2 \quad x \quad 10 \end{matrix}$, then

(a) Both f and g are functions
(b) f is not a function but g is
(c) f is a function but g is not
(d) Neither of them are functions

38. Let f be subset of $Z \quad Z$, where Z denotes the set of integers, and we have, $f \quad \{(ab, a \quad b): a, b \quad Z\}$. Then f, is

(a) injective function (b) surjective
(c) bijective function (d) None of these

39. For real numbers x, y and $n \quad Z$, $\sin x \quad \sin y$ implies which of the following ?

(a) $x \quad n \quad (\ \ 1)^n\ y$ (b) $x \quad 2n \quad y$
(c) $x \quad n \quad y$ (d) $x \quad n \quad y$

40. Show the bits in a 12 bit register that is holding the number equivalent to decimal 215 in (i) binary coded octal and (ii) binary coded decimal

(a) 000011010111, 001000010101
(b) 000011000111, 001000010111
(c) 000011000011, 001000010101
(d) None of the above

41. If $(24)^2 \quad 587$ the base of the number system, is

(a) 11 (b) 9 (c) 14 (d) 12

42. At how many points in the xy plane do the graphs of $y \quad x^{12}$ and $y \quad 2^x$ intersect?

(a) None (b) Two (c) One (d) Three

43. Suppose that f is a continuous real valued function defined on the closed interval $[0, 1]$. Which of the following are true for constants $C, D, E \quad 0$ and $x, y \quad [0, 1]$

I. There is C such that $|f(x) \quad f(y)| \quad C, \quad x, y$

II. There is D such that $|f(x) \quad f(y)| \quad 1, \quad x, y$ that satisfy $|x \quad y| \quad D$

III. There is E such that $|f(x) \quad f(y)| \quad E|x \quad y| \quad x, y$

(a) Only I (b) I and II
(c) Only III (d) I, II and III

44. If f is a function on the set of real numbers and is differentiable twice and that $f(0), f\ (0)$ and $f\ \ (0)$ are all negative. Suppose f has all the three properties in the interval $[0, \ \ ]$.

I. It is increasing II. It has a unique zero
III. It is unbounded

Which of the same three properties does f necessarily have?

(a) Only I (b) Only III
(c) Only II (d) II and III

45. Let $f(x) \quad 2x^3 \quad ax^2 \quad 3x \quad 5,$ and $g(x) \quad x^3 \quad x^2 \quad 4x \quad 9.$ Both $f(x), g(x)$ give the same reminder when divided by $x \quad 1$, if a is

(a) 1 (b)
(c) 10 (d) $\ \ 5$

46. Consider the unit square formed by points in the plane; $A(0, 0), B(1, 0), C(1, 1)$ and $D(0, 1)$. Let P be an arbitrary point chosen in this unit square. Connect P to the points A and B. The probability that the points ABP form an obtuse traingle is

(a) 0.393 (b) 0.712
(c) 0.5 (d) 0.25

47. Let f and g be two functions defined on an interval I such that $f(x) \quad 0$ and $g(x) \quad 0, \quad x \quad I$, and f is strictly increasing in I. Then, the product function fg, is

(a) strictly increasing in I with $fg \quad 0$
(b) strictly decreasing in I with $fg \quad 0$
(c) strictly increasing in I with $fg \quad 0$
(d) nothing can be said about it from the given information

48. If $[x]$ denotes the largest integer $\quad x$ and if $f(x) \quad x[x]$. $f\ \ (x)$ (where ever it exists) is given by?

(a) $2x$ (b) $2[x]$
(c) $[x]$ (d) exists nowhere

49. The sum of the interior angles of a polygon is equal to 86 right angles. Then, the number of sides of that polygon is

(a) 86 (b) 30 (c) 43 (d) 45

50. The area of the polygon whose vertices are $(x, x), (x \quad 1, x \quad 1)$ and $(x, x \quad 2)$ is equal to

(a) $x^2/2$ (b) 1 (c) $2x$ (d) 2

51. Suppose a, b and c are all $\quad 0$, and if D is the determinant below, which of the following is true about D ?

$$D \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

I. $D \quad 0$ II. $D \quad 0$ III. $D \quad 0$

(a) Only I (b) Both I and II
(c) Only II (d) Both II and III

52. If a, b and c are all $\quad 0$, and are in a geometric progression as the successive terms $\ldots,\ p, q, r,$ Then, what is the value of the determinant D below?

$$D \quad \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

(a) 0 (b) pqr (c) 1 (d) $pqr \quad abc$

53. Consider the determinant $\begin{vmatrix} 1 & \sin & 1 \\ \sin & 1 & \sin \\ 1 & \sin & 1 \end{vmatrix}$. Then, $\frac{}{2}$ for all values of , lies between

(a) 0 and 1 (b) 2 and 4
(c) 1 and 2 (d) 4

54. How many 2-digit or 3-digit numbers can be formed using the digits 1,3,4,5,6,8 and 9, which are divisible by 4?

(a) 56 (b) 80
(c) 64 (d) 92

55. If $(256)_{10} \quad (A)_8 \quad (B)_{16}$ and $(B)_{10} \quad 2 \quad (A)_{10}$, then the values of A, B are respectively

(a) $(7, 14)$ (b) $(15, 30)$
(c) $(6, 12)$ (d) $(10, 20)$

56. The average temperature of a town in the first four days of a month was 58 degrees. The average for the second, third, fourth, and fifth days was 60 degrees. If the temperatures of the first and fifth days were in the ratio 7 : 8, then the temperature on the fifth day was

(a) 64 (b) 56
(c) 62 (d) None of these

57. Two boys A and B speak the truth only 75% and 80% of the time, respectively. Lets say both witnessed an incident, what is the percentage of time that the two would contradict each other when narrating the same incident?

(a) 15% (b) 35%
(c) 25% (d) 45%

58. The solution of initial value problem $y \quad 5y \quad 6y \quad 0$, given $y(0) \quad 10$; $y\ (0) \quad 10$ is

(a) $y(t) \quad \frac{20}{7}e^{6t} \quad \frac{50}{7}e^{\ t}$

(b) $y(t) \quad \frac{20}{7}e^{\ 6t} \quad \frac{50}{7}e^{t}$

(c) $y(t) \quad 4e^{\ 6t} \quad 10e^{t}$

(d) $y(t) \quad 20e^{\ 3t} \quad 30e^{2t}$

59. When 4 dice are thrown, what is the probability that the same number appears on each of them?

(a) 1/1296 (b) 1/216 (c) 1/36 (d) 24/216

60. The mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19 and 21 is

(a) 1.01 (b) 9 (c) 5.27 (d) 10.01

61. Let $g(x) \quad {}_{10}^{\ x}\ tf\ (t)\,dt$ for $x \quad 10$, where f is an increasing function, then

(a) $g(x)$ is an increasing function of x
(b) $g(x)$ is increasing for $x \quad 0$, and decreasing for $10 \quad x \quad 0$
(c) $g(x)$ is a decreasing function of x
(d) None of the above

62. The solution to the indefinite integr

$\frac{\sqrt{a^2 \quad u^2}}{u^2}\ du$, is given by

(a) $2u^2a^2\ \sqrt{(a^2 \quad u^2)} \quad \sin^{\ 1}\frac{u}{a} \quad C$

(b) $\frac{1}{u}\sqrt{(a^2 \quad u^2)} \quad \cos^{\ 1}\frac{u}{a} \quad C$

(c) $\frac{1}{u}\sqrt{(a^2 \quad u^2)} \quad \sin^{\ 1}\frac{u}{a} \quad C$

(d) $2u^2a^2\sqrt{(a^2 \quad u^2)} \quad \sinh^{\ 1}\frac{u}{a} \quad C$

63. Consider the circles, all of unit radius given by the form $(x \quad h)^2 \quad (y \quad k)^2 \quad 1$. The radius of curvature at any point (x, y) for these circles may be also expressed as

(a) $1 \quad \frac{dy}{dx}^{2\ 3} \quad \frac{d^2y}{dx^2}^{2}$

(b) $1 \quad \frac{dy}{dx}^{2\ 3/2} \quad \frac{d^2y}{dx^2}^{3/2}$

(c) $1 \quad \frac{dy}{dx}^{2\ 2} \quad \frac{d^2y}{dx^2}^{3}$

(d) None of the above

64. The solution of differential equation $\frac{d^4y}{dx^4} \quad 4y \quad 0$ is

(a) $y \quad e^{x}(c_1 \cos x \quad c_2 \sin x) \quad e^x(c_3 \cos x \quad c_4 \sin x)$

(b) $y \quad e^{x}(c_1 \cos 2x \quad c_2 \sin 2x) \quad e^x(c_3 \cos 2x \quad c_4 \sin 2x)$

(c) $y \quad e^{2x}(c_1 \cos x \quad c_2 \sin x) \quad e^{2x}(c_3 \cos x \quad c_4 \sin x)$

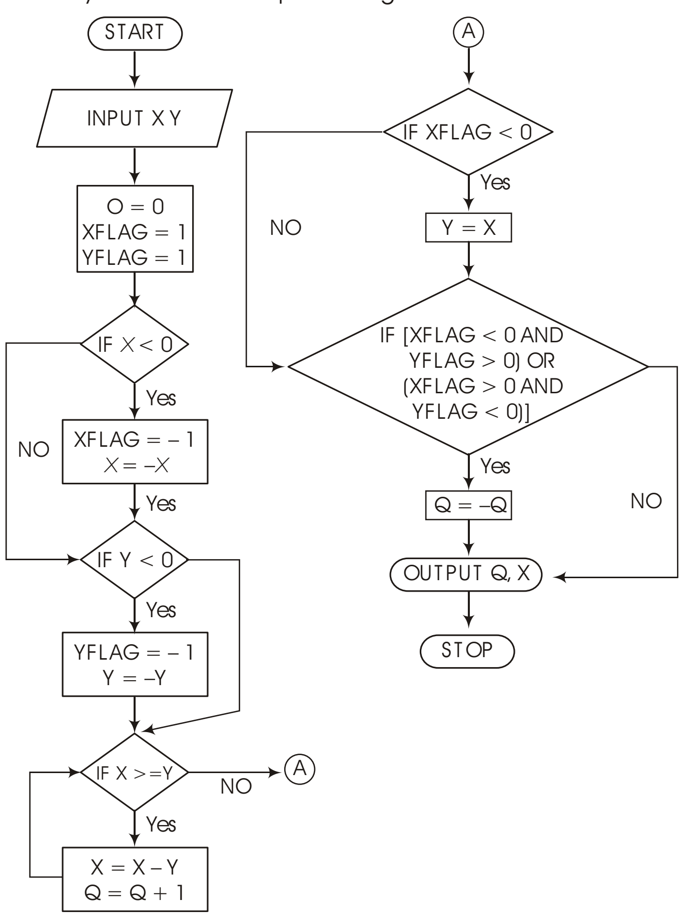
(d) $y \quad e^x(c_1 \cos x \quad c_2 \cos x) \quad e^x(c_3 \sin x \quad c_4 \sin x)$

65. The solution to the differential equation $(1 \quad xy)\, ydx \quad (1 \quad yx)\, xdy \quad 0$ is

(a) $\log \frac{x}{y} \quad \frac{1}{xy} \quad C$ (b) $\log \frac{y}{x} \quad xy \quad C$

(c) $\log \frac{x}{y} \quad xy \quad C$ (d) $\log \frac{y}{x} \quad \frac{1}{xy} \quad C$

Direction (Q. Nos. 66-68) study the following chart given carefully to answer the questions given below.



66. What is the output of the flow chart, if $X \quad 10$ and $Y \quad 3$?

(a) $Q \quad 3$ and $X \quad 1$ (b) $Q \quad 3$ and $X \quad 1$

(c) $Q \quad 3$ and $X \quad 1$ (d) $Q \quad 6$ and $X \quad 1$

67. Which of the following conditions can be used in place of the condition 'If, (XFLAG < 0 AND YFLAG > 0) or (XFLAG > 0 AND YFLAG < 0)'?

(a) If (XFLAG YFLAG 0) (b) If (XFLAG YFLAG 2)

(c) If (XFLAG YFLAG 2) (d) If (XFLAG YFLAG 0)

68. For what values of X and Y, the flow chart is never going to terminate?

(a) $X \quad 0$ and $y \quad 0$ (b) $X \quad 0$ and $Y \quad 0$

(c) $X \quad 0$ and $y \quad 0$ (d) None of these

69. A person speaks truth only 4 times out of 5. A die is tossed and the person says that the die rolled a six. Find the probability that actually there was a six.

(a) 2/9 (b) 7/9

(c) 4/5 (d) 4/9

70. $\cos A \quad \cos B$ can also be written as

(a) $2\cos \frac{(A \quad B)}{2} \cos \frac{(A \quad B)}{2}$

(b) $2\cos \frac{(A \quad B)}{2} \sin \frac{(A \quad B)}{2}$

(c) $2\cos \frac{(A \quad B)}{2} \sin \frac{(A \quad B)}{2}$

(d) $2\sin \frac{(A \quad B)}{2} \sin \frac{(A \quad B)}{2}$

71. The vectors $i \quad j \quad k, i \quad j \quad k$ and $2i \quad j \quad k$ are coplanar, if

I. $\quad 1$ II. $\frac{1 \quad \sqrt{5}}{2}$ III. $\quad 0$

Which of the following is correct?

(a) I and II (b) II and III

(c) I and III (d) Only I

72. If the vectors $a\mathbf{i} \quad \mathbf{j} \quad \mathbf{k}, \mathbf{i} \quad b\mathbf{j} \quad \mathbf{k}$ and $\mathbf{i} \quad \mathbf{j} \quad c\mathbf{k}$ (where a, b and $c \quad 1$) are coplanar, then $\frac{1}{1 \quad a} \quad \frac{1}{1 \quad b} \quad \frac{1}{1 \quad c}$ is equal to

(a) 1 (b) $\frac{1 \quad abc}{(1 \quad a)(1 \quad b)(1 \quad c)}$

(c) $\frac{1}{abc}$ (d) 0

73. If $\frac{2\sin}{1 \quad \cos \quad \sin} \quad y$, then $\frac{1 \quad \cos \quad \sin}{1 \quad \sin}$ is equal to

(a) $\frac{1}{y}$ (b) $1 \quad y$ (c) y (d) $1 \quad y$

74. Which of the following is equal to the nullity of matrix A below?

$$A \quad \begin{matrix} 1 & 4 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 0 \\ 2 & 6 & 4 \end{matrix}$$

(a) 0 (b) 2 (c) 1 (d) 3

75. The value of the expression

$$1 \quad \frac{\cos^2 x}{1 \quad \sin x} \quad \frac{1 \quad \sin x}{\cos x} \quad \frac{\cos x}{1 \quad \sin x}$$ is equal to

(a) 0 (b) $\cos x$ (c) $\sin x$ (d) 1