

**(June 2007)**

1. The English words for two numbers are scrambled and the letters reassembled to give two different words for two different numbers. When these two numbers are added together, it is found that their sum is the same as that of the original numbers. One of the four numbers is  
(a) 11 (b) 13  
(c) 5 (d) 7  
(e) None of the above
2. The minimum number of weights needed to weigh objects ranging from 1 kg. to 364kg. in a two-pan balance is  
(a) 6 (b) 5  
(c) 9 (d) 8  
(e) None of the above
3. What is the smallest 3-digit integer number that can be expressed as the sum of two squares in three different ways :  
(a) 625 (b) 325  
(c) 425 (d) 369  
(e) None of the above.
4. Which of the following 10-digit numbers containing each digit once so that the number formed by the first  $n$  digits is divisible by  $n$  for each value of  $n$  between 1 and 10 ?  
(a) 3816574290 (b) 24  
(c)

# 2007

## Part B

26. Let A be a set with 10 elements. The total number of relations that can be defined on A which are both reflexive and symmetric is  
 (a)  $2^{45}$  (b)  $2^{55}$   
 (c)  $10^{55}$  (d) None of the above
27. Two classes A and B with respective strengths of 30 and 40 have an arithmetic mean of 50 and 60 respectively and standard deviation of 2 and 4 respectively. The respective group mean and standard deviation of the two classes are :  
 (a) 55.7, 3.3 (b) 55.0, 3.0  
 (c) 55.0, 3.3 (d) 55.7, 3.0  
 (e) None of the above (Hyd.-2007)
28. An  $n \times m$ -matrix A is of rank 5, and  $5 < n$  and  $5 < m$ . Identify the statement which is NOT TRUE.  
 (a) All the determinants of sub-square matrix of order  $k \times k$  ( $k < 5$ ) are zero  
 (b) There exists a set of  $k$  rows ( $k < 5$ ) which are linearly independent  
 (c) The set of any  $k$  ( $k > 5$ ) rows are linearly dependent  
 (d) All the determinants of sub-square matrix of order  $k \times k$  ( $k > 5$ ) are zero  
 (e) There exists a  $5 \times 5$  submatrix whose determinant is non-zero
29. Which of the following statements is TRUE?  
 (a) if a relation is not reflexive then it is irreflexive  
 (b) if a relation is irreflexive then it is asymmetric  
 (c) if a relation is asymmetric then it is irreflexive  
 (d) if a relation is not asymmetric then it is asymmetric  
 (e) None of the above
30. If \_\_\_\_\_ then at \_\_\_\_\_  
 (a) 0 (b)  $\infty$   
 (c) 1 (d)  $-\infty$   
 (e) None of the above
31. The mean (m) and standard deviation (sd) of the binomial distribution with  $n = 48$  and  $p = 0.75$  is  
 (a)  $m = 36, sd = 9$  (b)  $m = 12, sd = 9$   
 (c)  $m = 12, sd = 3$  (d)  $m = 36, sd = 3$   
 (e) None of the above
32. A function  $f$  is defined on the whole of R as follows: for some  $k$  ( $0 < k < 1$ ),  
 \_\_\_\_\_  
 Then \_\_\_\_\_ is \_\_\_\_\_  
 (a) 1 (b) 0  
 (c)  $\infty$  (d)  $-\infty$   
 (e) None of the above
33. The number  $2^{300} * 5^{600} * 4^{400}$  ends in how many zeros?  
 (a) 300 (b) 600  
 (c) 400 (d) 700  
 (e) None of the above
34. A set A contains  $(2n + 1)$  elements. The number of subsets of A which contain at most  $n$  elements is  
 (a)  $2^n$  (b)  $2^{n+1}$   
 (c)  $2^{n-1}$  (d)  $2^{2n}$   
 (e) None of the above
35. Let  $A = \{p, q, r, s\}$  and  $B = \{1, 2, 3, 4, 5, 6\}$ . How many one-to-one functions are possible from A to B?  
 (a)  $4^6$  (b)  $6^4$   
 (c) 24 (d) 360  
 (e) None of the above
36. Set A contains 3 elements and set B contains 4 elements. The number of onto functions from A to B is  
 (a) 2 (b) 3  
 (c) 6 (d) 5  
 (e) None of the above
37. Let A, B, C be disjoint subsets of a sample space S and E, F form a partition of S. Let  $P(A) = a, P(B) = b, P(C) = c, P(E) = e, P(F) = f, P(A|E) = x, P(A|F) = y$ . Given that  
 I.  $a + b + c = 1$   
 II.  $e + f = 1$   
 III.  $aP(E|A) = xe$   
 (a) II and I (b) III and I  
 (c) II and III (d) I only  
 (e) None of the above
38. If \_\_\_\_\_ then  $A^6$  is  
 (a) A (b)  $A^2$   
 (c)  $A^3$  (d) I  
 (e) None of the above
39. The value of a for which the system of equations  
 $a^3x + (a + 1)^3y + (a + 2)^3z = 0$   
 $ax + (a + 1)y + (a + 2)z = 0$   
 $x + y + z = 0$  has a non-zero solution is  
 (a) 1 (b) -1  
 (c) 0 (d) 2  
 (e) None of the above
40. If \_\_\_\_\_ then A cannot be  
 (a) Symmetric (b) Skew-symmetric  
 (c) Hermitian (d) Idempotent  
 (e) None of the above
41. If 1 and 2 are the eigen values of 2rowed square matrix A and  $I_2$  is the unit matrix of order 2, then  $A^3$  is equal to  
 (a)  $6A + I_2$  (b)  $6A - I_2$   
 (c)  $7A + 6I_2$  (d)  $7A - 6I_2$   
 (e) None of the above
42. A and B are  $3 \times 3$  matrices and A is invertible. Let eigen values of AB be  $\lambda_1, \lambda_2$  and  $\lambda_3$ . Then, eigen values of BA are

- (a) 1, 1, 1, (b)  $1/1, 1/1, 1/1_3$   
(c) (d)  
(e) None of the above
43. A man has seven relatives. Four of them are ladies and three are gentlemen. His wife has seven relatives. Three of them are ladies and four are gentlemen. In how many ways can they throw a dinner party of three ladies and three gentlemen so that there are three of husband's relatives and three of wife's relatives ?  
(a) 485 (b) 465  
(c) 475 (d) 490  
(e) None of the above
44. What are the values of  $\$, \otimes$  and  $\pi$  such that  $\$ + \$ + \$ + \otimes = \$ + \$ + \otimes + \otimes + \otimes = \pi + \pi$  and  $\pi - \$ = 6$  ?  
(a)  $\pi = 14, \$ = 8, \otimes = 4$  (b)  $\pi = 41, \$ = 18, \otimes = 4$   
(c)  $\pi = 14, \$ = 8, \otimes = 14$  (d)  $\pi = 14, \$ = 18, \otimes = 4$   
(e) None of the above
45. Imagine a  $3 \times 3 \times 3$ -inch opaque cube divided into 27 1-inch cubes. What is the maximum number of 1-inch cubes that can be seen by one person from any point in space ?  
(a) 9 (b) 19  
(c) 18 (d) 6  
(e) None of the above
46. In the expression below, what digits do the three letters represent ?  
 $(MCCA)_{base8} - (MCCA)_{base5} = (MCCA)_{base7}$   
(a)  $M = 2, C = 1, A = 1$  (b)  $M = 1, C = 3, A = 2$   
(c)  $M = 2, C = 3, A = 1$  (d)  $M = 5, C = 2, A = 1$   
(e) None of the above
47. The nullity of the matrix is  
(a) 3 (b) 2  
(c) 1 (d) 0  
(e) None of the above
48. If  $0 \leq \theta \leq 90$  such that  $\tan \theta =$   
then, is equal to  
(a)  $231 / 377$  (b)  $231 / 377$   
(c)  $231 / 337$  (d)  $213 / 377$   
(e) None of the above
49. The angle of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 KM apart is  $45^\circ$  and  $60^\circ$ . The height of the aeroplane above the ground in KM is :  
(a) (b)
- (c) (d)  
(e) None of the above
50. The angle of elevation of the top of a tower as observed from a point on the horizontal ground is  $x$ . If we move a distance  $d$  towards the foot of the tower, the angle of elevation increases to  $y$ . The height of the tower is :  
(a) (b)  $d(\tan y - \tan x)$   
(c)  $d(\tan x - \tan y)$  (d)  
(e)
51. Solution for the first order differential equation is such that  
(a) The solution surface is exponential in both  $x$  and  $y$   
(b) The solution surface is exponential in  $x$   
(c) The solution surface passes through the origin  
(d) The solution surface is exponential in  $y$   
(e) None of the above
52. If  $f(x)$  is a polynomial satisfying  $f(x) \cdot f(1/x) = f(x) + f(1/x)$  and  $f(3) = 28$ , then  $f(4)$  is given by  
(a) 63 (b) 67  
(c) 65 (d) 68  
(e) None of the above
53.  $f(x) = x^2 + 1, g(x) = 2x$  and  $h = fog(x)$ . Then the following statement is NOT TRUE.  
(a) Minimum value of  $h$  is 1  
(b)  $h(x)$  is always positive  
(c)  $h^{-1}(17) = \{2\}$   
(d)  $h(x)$  is symmetric about  $y$ -axis  
(e) The graph of  $h(x)$  does not touch the line  $y = x$
54. The volume  $V$  of the tetrahedron with  $a = [2, 0, 3], b = [0, 6, 2], c = [3, 3, 0]$  as edge vectors is  
(a) 10 (b) 12  
(c) -66 (d) 11  
(e) None of the above
55. The angle between the resultant  $r$  of the forces  $a = [3, 2, 0]$  and  $b = [-1, 4, 0]$  and the  $x$ -axis is  
(a)  $\arccos 0.31623$  (b)  $\arcsin 0.31623$   
(c)  $\arccos 1.632$  (d)  $\arcsin 1.632$   
(e) None of the above
56. If = then  
(a)  $f$  is continuous at  $x = -2$   
(b)  $f$  is differentiable at  $x = -2$   
(c)  $f$  is neither differentiable nor continuous at  $x = -2$   
(d)  $f$  is continuous at  $x = -2$  but not differentiable  
(e) None of the above
57. Which of the following statements is true given that the conditional probabilities are equal :  $\Pr(A|B) =$

$\Pr(B|A)$

- (a)  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
- (b)  $\Pr(A \cap B) = \Pr(A) * \Pr(B)$
- (c)  $\Pr(A) = \Pr(B)$
- (d)  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A) * \Pr(B)$
- (e) None of the above

58. A box has 3 black and 4 red balls. A ball is drawn (first draw) at random. Two new balls of the same color as the first drawn ball are dropped into the box. A ball is then drawn at random (second draw). What is the probability of the color of the second drawn ball being black ?

- (a)  $12 / 63$
- (b)  $15 / 63$
- (c)  $18 / 63$
- (d)  $27 / 63$
- (e) None of the above

59. The geometric mean of the roots of a sixth degree polynomial  $P(x)$  with  $P(0)=729, P(1)=P(2.846)=P(3)=P(5) = P(6) = 0$  is

- (a) 6
- (b) 2
- (c) 3
- (d) 4
- (e) inadequate information

60. The following is what seems to be a mathematical proof that 10 equals 9.999999... What's wrong with it ?

$a = 9.999999$   
 $10a = 99.999999$   
 $10a - a = 90$   
 $9a = 90$   
 $a = 10$

- (a) rounding of error
- (b) approximation error
- (c) rationalization error
- (d) nothing wrong
- (e) None of the above

61. The curve  $y = ax^3 + bx^2 + cx + 5$  touches the x-axis at  $P(-2, 0)$  and cuts the y-axis at a point Q where its gradient is 3. The values of a, b, c are

- (a)  $a = 1/2, b = 3/4, c = 3$
- (b)  $a = -1/2, b = -3/4, c = 3$
- (c)  $a = -1/2, b = -3/4, c = -3$
- (d)  $a = -1/2, b = 3/4, c = 3$
- (e) None of the above

62. How many times on average must a six-sided dice be tossed before every number from one to six comes up at least once ?

- (a) 14
- (b) 13.7
- (c) 12
- (d) 14.7
- (e) None of the above

63. If A, B are two matrices and  $AB = 0$ , it implies

- (a)  $BA = 0$
- (b) A and B are orthogonal matrices
- (c) Definitely either  $A = 0$  or that  $B = 0$
- (d) Does not imply either  $A = 0$  or  $B = 0$
- (e) None of the above

64. The following statement is NOT TRUE:

- (a) The function  $\tan x$  is one-to-one from  $[-1, 1]$  to  $\mathbb{R}$
- (b) The function  $\tan x : [-1, 1] \rightarrow \mathbb{R}$  is not onto
- (c) The function  $\tan x$  and  $\sin x$  intersect at infinitely many points along the real line

(d)  $\tan x$  is a periodic function of period  $2\pi$

(e)  $\tan x$  is asymptotic to  $x =$  for all  $n \in \mathbb{N}$

65. The result of the subtraction  $FD_{16} - 88_{16}$  in base 2 is

- (a) 01110101
- (b) 10011001
- (c) 01101001
- (d) 10011110
- (e) None of the above

66. If \_\_\_\_\_ the value of \_\_\_\_\_

at  $x = 0$  is

- (a) -1
- (b) 0
- (c) 1
- (d) undefined
- (e) None of the above

67. If  $a^1 p, b^1 q, c^1 r$  and \_\_\_\_\_ then the value \_\_\_\_\_

of \_\_\_\_\_ is

- (a) 2
- (b) 0
- (c) 1
- (d) 3
- (e) -2

68. Let X be a set containing n elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_
- (d) \_\_\_\_\_
- (e) None of the above

69. Suppose m boys and m girls take their seats randomly round a circle. In the following situation, the

probability of their seating arrangement is

- (a) When two boys sit together
- (b) When two girls sit together
- (c) When boys and girls sit alternately
- (d) When m girls and one boy sit together
- (e) None of the above

70. If A and B are acute positive angles satisfying the equation  $3 \sin^2 A + 2 \sin^2 B = 1$  and  $3 \sin 2A - 2 \sin 2B = 0$ , then  $A = 2B =$

- (a)  $\pi$
- (b) \_\_\_\_\_

- (c) \_\_\_\_\_
- (d) \_\_\_\_\_

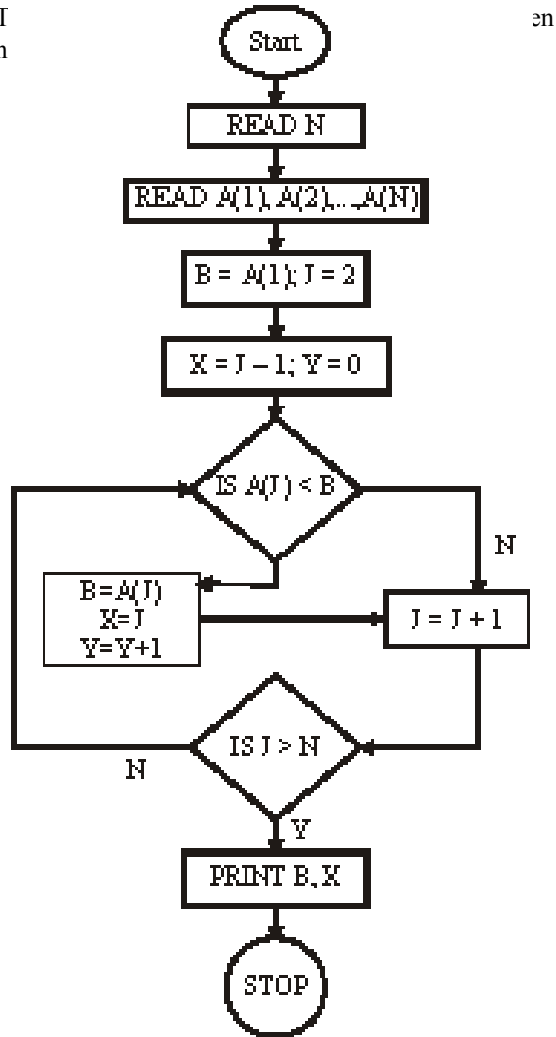
(e) None of the above

71. If p and q are chosen randomly from the set  $\{1, 2, 3, 4, 5, 6\}$  with replacement, the probability that

the roots of the equation  $x^2 + px + q = 0$  are real is :

- (a) 0.53 (b) 0.25  
 (c) 0.75 (d) 0.3  
 (e) None of the above

T  
h



(e) All of the above are false

75. Suppose the input array is [2, 2, 2, 2, 1, 1, 1, 1, 0, 0, 0, 0, 3, 3, 3, 3] then, the output of the flow chart is

- (a) 0, 0 (b) 0, 12  
 (c) 0, 9 (d) 0, 16  
 (e) None of the above

72. On the input array [3, 1, 2, 1, 4, 3, 1, 100] at the instance when  $j = 4$  the value that B holds is  
 (a) 2 (b) 1  
 (c) 4 (d) 3  
 (e) None of the above
73. On any input array, the number of times that the value of B gets updated is equal to  
 (a) X (b) Y  
 (c) J (d)  $Y - 1$   
 (e) None of the above
74. Which of the following statements is TRUE ?  
 (a) If the array contains distinct elements, then, the value of Y is maximum  
 (b) If the array contains identical elements, then, the value of Y is minimum  
 (c) If the value of Y is 0, then the array contains identical elements  
 (d) The value of X denotes the number of updations on B































